Dec 2012

This question paper contains 2 printed pages. Your Roll No. ....

18

Sl. No. Of Ques. Paper	:	8381C
Unique Paper Code	:	222101
Name of the Paper	:	PHHT- 101 : Mathematical Physics - I
Name of the Course	:	B.Sc. (Hons) Physics Part I
Semester	:	I
Duration	:	3 hours
Maximum Marks	:	75

Attempt five questions in all. Question No. 1 is compulsory.

Q 1.

a) Explain the physical significance of the curl of a vector field. **(3)**

b) Differentiate between systematic and random errors. **(3)**

c) A point has cartesian coordinates as (4, 3, 12). Write an expression for the position vector in cylindrical and spherical coordinates. **(3)**

d) Prove that $\sin x$ and $\cos x$ are orthogonal in the interval $(0, 2\pi)$. **(3)**

e) Evaluate the integral $\int_0^1 \left(\log\frac{1}{x}\right)^{n-1} dx, \quad n > 0$. **(3)**

Q 2.

a) Prove that a spherical coordinate system is orthogonal. **(4)**

b) Derive the expression for the divergence of a vector field in curvilinear coordinates and express it in spherical coordinates. **(4+3)**

c) Evaluate $\iiint_V \sqrt{(x^2+y^2)}\,dxdydz$, where V is the region bounded by $z = x^2 + y^2$ and $z = 8 - x^2 - y^2$. **(4)**

Q 3.

a) Verify Green theorem in the plane for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary of the region defined by $x = 0, y = 0, x + y = 1$. **(3+2)**

b) State and prove Stokes' theorem. **(2+5)**

c) Let $f = x^2yz - 4xyz^2$ be a scalar field. Find the directional derivative of f at P (1, 3, 1) in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$. Also find the magnitude of maximum directional derivative. **(2+1)**

Q 4.

a) Prove the identity $\vec{\nabla}\cdot(\vec{A}\times\vec{B})=\vec{B}\cdot(\vec{\nabla}\times\vec{A})-\vec{A}\cdot(\vec{\nabla}\times\vec{B})$. **(5)**

b) Evaluate $\nabla^2\left[\vec{\nabla}\cdot\frac{\vec{r}}{r^2}\right]$, where $\vec{r}=x\hat{i}+y\hat{j}+z\vec{k}$. **(6)**

c) Assuming that $f(r)$ is differentiable, prove that $f(r)\vec{r}$ is irrotational. **(4)**

Q 5.

a) Find the Fourier series expansion of the output of a half wave rectifier. Draw the rectified wave function between 0 to T, where T is the time period. **(7+2)**

b) Obtain a cosine series expansion of the function $f(x)=1+x$ valid in the interval $0\le x\le 2$ and hence sum the series $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\cdots$. **(3+1)**

c) State the Dirichlet theorem so that a function can be expressed as a Fourier series. **(2)**

Q 6.

a) Prove the identity $\frac{\Gamma m\,\Gamma n}{\Gamma(m+n)}=B(m,n)$, where all the symbols have usual meaning. **(8)**

b) State the law of propagation of errors. The resistances of two resistors were determined several times giving the results $R_1=(3.52\pm 0.01)\Omega$ and $R_2=(5.12\pm 0.01)\Omega$. Calculate standard error in the total resistance R in series and in parallel. **(1+2+4)**

Q 7.

a) Prove that $\vec{\nabla}\left[\vec{M}\cdot\vec{\nabla}\left(\frac{1}{r}\right)\right]=\frac{3\left[\vec{M}\cdot\vec{r}\right]\vec{r}}{r^5}-\frac{\vec{M}}{r^3}$ **(6)**

b) Show that $B(m,n)=2\int_0^{\pi/2}(\sin\theta)^{2m-1}(\cos\theta)^{2n-1}\,d\theta$ **(4)**

c) Express the position and the velocity vectors of a particle in cylindrical coordinates. **(2+3)**

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 7876 2013

Unique Paper Code : 222101 F-1

Name of the Paper : **Mathematical Physics-I (PHHT-101)**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all including Q. No. 1 which is compulsory.

1. Do any *five* of the following : 5×3=15

(*a*) Define a Jacobian. Calculate the Jacobian for a change from cartesian (x, y) to polar coordinates (r, θ) in two dimensions.

(*b*) What is Wronskian ? Calculate the Wronskian of x^n and $x^n \log x$.

(*c*) Find the minimum value of $u = x^4 + y^4 + z^4$ on the surface $xyz = c^3$, where c is a constant.

(*d*) Establish :

$$\delta(x^2 - a^2) = \frac{\delta(x-a) + \delta(x+a)}{2|a|}$$

(*e*) Show that the force :

$$\vec{F} = r^2 \vec{r}$$

is conservative.

P.T.O.

(*f*) Evaluate $\vec{\nabla}\left(\vec{F}\cdot\vec{r}\right)$, where $\vec{F}$ is a constant vector.

(g) Solve :

$$\cos(x+y)\,dy = dx$$

2. (*a*) Solve the following differential equations :

(*i*) $\sec^2 y \dfrac{dy}{dx} + 2x\tan y = x$

given that $y(1) = \dfrac{\pi}{4}$

(*ii*) $4\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + y = x(x + e^{-x/2})$ 5,5

(*b*) Solve the differential equation : 5

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x\log x$$

3. (*a*) Find the particular integral of the following differential equation : 5

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = \frac{5}{4}e^{x/2} + 18\cos 4x - 71\sin 4x$$

(*b*) Solve : 5

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4$$

(*c*) Show that the following equation is exact and then solve it : 5

$$\{(x+1)e^x - e^y\}\,dx = xe^y\,dy$$

4. (a) Evaluate the gradient ϕ, where ϕ is defined by :

$$\phi = \frac{\vec{p} \cdot \vec{r}}{r^3}$$

where $\vec{p}$ is a constant vector.

(b) Prove : 5

$$\frac{1}{2} \vec{\nabla} \times \left(\vec{\omega} \times \vec{r} \right) = \vec{\omega}$$

(c) Derive an expression for the divergence of $\vec{A}$ in spherical coordinates. 6

5. (a) Find the unit normal to the surface : 5

$$Z = \sqrt{\frac{3}{2}x^2 + \frac{3}{2}y^2}$$

at the point $\left(\sqrt{\frac{2}{3}}, 0, 1 \right)$.

(b) Which the following is not solenoidal ?

(i) $\dfrac{B_0 r_0^2 \left(x\hat{i} + y\hat{j} + z\hat{k} \right)}{\left(x^2 + y^2 + z^2 \right)^{3/2}}$

(ii) $B_0 \left(\dfrac{yz\hat{i}}{x^2 + y^2} - \dfrac{xz\hat{j}}{x^2 + y^2} \right)$

where B_0 and r_0 are constants. 4

(c) Obtain an expression for the curl of $\vec{A}$ in cylindrical coordinates. 6

6. (*a*) State Green's theorem in the plane. Verify Green's theorem in the plane for : 8

$$\oint_C [3x^2 - 8y^2]\, dx + [4y - 6xy] dy$$

where C is the boundary defined by :

$$x = 0,\ y = 0,\ x + y = 1.$$

(*b*) State the divergence theorem. Verify the divergence theorem for the vector field :

$$\vec{A} = 2x^2 y\hat{i} - y^2\hat{j} + 4xz^2\hat{k}$$

taken over the region in the first octant bounded by : 7

$$y^2 + z^2 = 9, x = 2.$$

7. (*a*) State Stokes' theorem. Verify Stokes' theorem for the vector field :

$$\vec{A} = (2x - y)\hat{i} - yz^2\hat{j} - y^2 z\hat{k}$$

where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. 8

(*b*) Evaluate :

$$\iint_S \vec{A} \cdot \hat{n}\, dS \text{ where } \vec{A} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$$

and S is that part of the plane $2x + 3y + 6z = 12$ which is located in the first octant and $\hat{n}$ is the unit outward normal to S. 7

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Roll No. | | | | | | | | | | |

S. No. of Question Paper : **6201**

Unique Paper Code : **222101** **D**

Name of the Paper : **Mathematical Physics-I (PHHT-101)**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all.

Question No. **1** is compulsory.

1. Do any *five* parts : 5×3=15

(*a*) Prove that :

$$\vec{\nabla} \times \left(\vec{\nabla} \phi \right) = 0.$$

(*b*) Define Jacobian. Calculate the Jacobian for a change from Cartesian (x, y) to polar coordinates (r, θ)

(*c*) What is the physical significance of precision constant 'h' ? Which *one* of the two sets of data having $h = 7$ and $h = 7.5$ respectively will have better precision ?

P.T.O.

(*d*) Prove that $\sin x$ and $\cos x$ are orthogonal in the interval $(0, 2\pi)$.

(*e*) State Dirichlet's conditions for Fourier series.

(*f*) Find the value of :

$$\beta\left(\frac{3}{2}, 2\right).$$

(g) Find the unit outward drawn normal to the surface :

$$(x - 1)^2 + y^2 + (z + 2)^2 = 9$$

at the point (3, 1, –4).

2. (*a*) Prove that :

$$\vec{\nabla} \cdot \left(\phi \vec{A}\right) = \left(\vec{\nabla}\phi\right) \cdot \vec{A} + \phi\left(\vec{\nabla} \cdot \vec{A}\right).$$ 5

(*b*) Show that :

$$\vec{A} = (x + 2y + 4z)\,\hat{i} + (2x - 3y - z)\,\hat{j} + (4x - y + 2z)\,\hat{k}$$

is irrotational. Find ϕ such that $\vec{A} = \vec{\nabla}\phi$. 5

(*c*) Evaluate :

$$\nabla^2(\ln r).$$ 5

3. (*a*) State and prove Gauss's Divergence theorem. 10

(*b*) If

$$\vec{F} = y\hat{i} + (x - 2xz)\hat{j} - xy\hat{k},$$

evaluate :

$$\iint_S \left(\vec{\nabla} \times \vec{F}\right) \cdot \hat{n}\, dS$$

where S is the surface of sphere $x^2 + y^2 + z^2 = a^2$ above the xy plane. 5

4. (*a*) Obtain an expression for the divergence of a vector in orthogonal curvilinear coordinates and express it in cylindrical coordinates. 8

(*b*) Evaluate using Green's theorem :

$$\int x^2 y\, dx + y^3\, dy$$

where C is rectangular curve formed by joining the points (0, 0), (1, 0), (1, 1) and (0, 1). 7

5. (*a*) Find the volume of the region common to the intersecting cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$. 8

(*b*) Represent the vector :

$$\vec{A} = 2y\hat{i} - z\hat{j} + 3x\hat{k}$$

in cylindrical coordinates and determine A_ρ, A_θ and A_z. 7

6\. (a) Expand in a Fourier series :

$$f(x) = \begin{cases} \sin x & 0 \le x \le \pi \\ -\sin x & -\pi \le x \le 0 \end{cases}$$ 7

(b) Find the Fourier series for the periodic function $f(x)$ defined by :

$$f(x) \begin{cases} = -\pi & -\pi < x < 0 \\ = x & 0 < x < \pi \end{cases}$$

and deduce :

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \text{--------} = \frac{\pi^2}{8}.$$ 8

7\. (a) The radius of a wire is measured in cm as :

0.17, 0.15, 0.18, 0.19, 0.16, 0.17.

Find the mean radius and the standard error. 4

(b) Prove that :

$$\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}.$$ 6

(c) Evaluate :

$$\Gamma\left(\frac{1}{2}\right).$$ 5

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 906 28 NOV 2014

Unique Paper Code : 222101 E

Name of the Paper : **Mathematical Physics-I (PHHT-101)**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all.

Question No. **1** is compulsory.

1. Attempt any *five* from the following : 5×3=15

(*a*) Determine whether the force field :

$$\vec{F} = 2xz\hat{i} + (x^2 - y)\hat{j} + (2z - x^2)\hat{k}$$

is conservative or non-conservative ?

(*b*) Find the unit vector normal to the surface $x^2 + y^2 + z^2 = 4$ at the point $(1, \sqrt{2}, -1)$.

(*c*) Evaluate :

$$\int_0^\infty x^3 e^{-x} dx.$$

(*d*) State the Dirichlet conditions for Fourier series.

(*e*) Prove that the cylindrical co-ordinates system is orthogonal.

P.T.O.

(*f*) Evaluate :

$$J\left(\frac{u, v}{x, y}\right)$$

where $u = x^2$ and $v = y^2$.

(*g*) Show that area bounded by a simple closed curve C is given by :

$$\frac{1}{2}\oint xdy - ydx.$$

5

2. (*a*) Prove :

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = -\nabla^2 \vec{A} + \vec{\nabla}(\vec{\nabla} \cdot \vec{A}).$$

(*b*) Find Divergence and Curl of $\vec{A} = y^2\hat{i} + 2xy\hat{j} - z^2\hat{k}$ at the point (1, 2, 1). Is this vector : 5

(*i*) solenoidal

(*ii*) irrotational.

5

(*c*) Evaluate :

$$\vec{\nabla} \cdot (r^3 \vec{r}).$$

10

3. (*a*) State and prove Stokes' theorem.

5

(*b*) Evaluate :

$$\iint_S \vec{A} \cdot \hat{n}\, ds$$

where

$$\vec{A} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$$

and S is that part of the plane $2x + 3y + 5z = 12$ which is located in first octant.

4. (*a*) Evaluate : 8

$$\oint_C (x^2 - 2xy)\, dx + (x^2 y + 3) dy$$

Using Green's theorem where C is the boundary of the region enclosed by $y^2 = 8x$ and $x = 2$.

(*b*) Obtain an expression for the Curl of a vector in orthogonal curvilinear co-ordinates and express it in spherical co-ordinates. 7

5. (*a*) Evaluate : 7

$$\iiint_V (x^2 + y^2)^{1/2}\, dx\, dy\, dz$$

where V is volume of region bounded by :

$$z = x^2 + y^2,\ z = 8 - (x^2 + y^2).$$

(*b*) Express the vector : 8

$$\vec{A} = z\hat{i} - 2xy\hat{j} + y\hat{k}$$

in cylindrical co-ordinates and determine A_ρ, A_θ, and A_z.

6. (*a*) Find the Fourier Series expansion of the function $f(x) = x^2, -\pi \le x \le \pi$. Hence find the sum of series : 8

$$\frac{1}{1^2} - \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2} - ---$$

(*b*) Expand as Fourier series. The function :

$$f(x) = \begin{cases} -K & -\pi < x < 0 \\ +K & 0 < x < \pi \end{cases}$$

Hence find the sum

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} - - - - - -$$

7. (*a*) The diameter of a wire is measured in cm as :

0.28, 0.26, 0.24, 0.29, 0.27.

Find the mean radius and the standard error.

(*b*) Prove that :

$$\beta(m, n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}.$$

(*c*) Evaluate :

$$\int_0^\infty \sqrt{x}e^{-x}\,dx.$$

This question paper contains 3 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper	: 5756	F
Unique Paper Code	: 222101	
Name of Paper	: PHHT–101: Mathematical Physics – I	
Name of Course	: B.Sc. (Hons.) Physics	
Semester	: I	
Duration	: 3 hours	Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt five questions in all including Q. No. 1 which is compulsory.

1. Do any *five* of the following : 5 x3 =15

(a) Find the unit normal to the surface $x^2 + y^2 = z$ at a point (1,2,5).

(b) Show that $\vec{\nabla} r^n = nr^{n-2}\vec{r}$.

(c) Evaluate

$$\iint_S \vec{r}.\hat{n}\, dS$$

where S is surface of a unit sphere.

(d) Find $J\left(\frac{x,y}{u,v}\right)$, given $x=u^2 - v^2$ and $y = 2uv$.

(e) If $\vec{v} = \vec{\omega} X \vec{r}$, prove that $\vec{\omega} = \frac{1}{2} curl \vec{v}$, where $\vec{\omega}$ is a constant vector.

(f) State Normal Law of errors.

(g) Graph the following function and classify it as odd, even or neither

P.T.O

$$f(t) = \begin{cases} 0 & if \quad -\frac{T}{2} < t < 0 \\ 2k & 0 < t < T/2 \end{cases}$$

2. (a) Show that $\vec{F} = (2xy + z^3)\hat{\imath} + x^2\hat{\jmath} + 3xz^2\hat{k}$, is a conservative force field. Find the scalar potential. Also find the work done in moving an object in this field from (1, -2,1) to (3,1,4). (7)

   (b) Prove that $\vec{\nabla} X (\vec{\nabla} X\vec{A}) = \vec{\nabla}(\vec{\nabla}.\vec{A}) - \nabla^2\vec{A}$ (5)

   (c) Prove that $\vec{\nabla} X [f(r)\vec{r}] = 0$ (3)

3. (a) State and prove Gauss theorem. (8)

   (b) Evaluate $\iiint(2x+y)\, dV$ where V is the closed region bounded by the cylinder $z = 4-x^2$ and the planes x=0, y=0, y=2 and z=0. (7)

4. (a) Verify Green's theorem in the plane for $\int(3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary of the region defined by: $y = \sqrt{x}, y = x^2$ (10)

   (b) Find the surface area of the plane $x + 2y + 2z=12$ cut off by the planes x = 0, y = 0, x =1 and y=1. (5)

5. (a) Express $\vec{\nabla} X\vec{A}$ in an orthogonal curvilinear coordinates system. (5)

   (b)Evaluate $\iiint (x^2 + y^2 + z^2)\ dV$, where V is volume of the sphere $x^2 + y^2 + z^2 = a^2$. (5)

   (c) Express the position, velocity and acceleration vectors of a particle in cylindrical coordinates. (5)

6. (a) Find the Fourier series expansion for the following function of period 2π
   $f(x) = x^2/4 \quad (-\pi < x < \pi)$.

   Show that $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} \ldots\ldots\ldots. = \frac{\pi^2}{6}$ (8)

(b) If $f(x) = \begin{cases} x & 0 \leq x \leq \frac{\pi}{2} \\ \pi - x & \frac{\pi}{2} \leq x \leq \pi \end{cases}$

Graph the function and express the function as sine series. (7)

7. (a) Evaluate

$$\frac{\Gamma(5)\Gamma(3\cdot5)}{\Gamma(2\cdot5)}$$ (5)

(b) Prove $\int_0^{\frac{\pi}{2}} \sqrt{\tan\theta} = \frac{\pi}{\sqrt{2}}$ (5)

(c) The length of a cylinder when measured yields the following values:

4.19, 4.21, 4.23, 4.18, 4.16 and 4.20(in cm). Find the mean length and its standard error. (5)

450

This question paper contains 3 printed pages. 26 NOV 2015 Your Roll No. ....

Sl. No. of Ques. Paper : 6551 FC

Unique Paper Code : 32221101

Name of Paper : Mathematical Physics – I

Name of Course : B.Sc. (Hons.) Physics : Choice-based credit system

Semester : I

Duration : 3 hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt five questions in all.

1. a) By calculating the Wronskian of the functions -1, sin^2x, cos^2x, check whether the functions are linearly dependent or independent. 4

   b) Solve the inexact equation

$$y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$$
5

   c) Solve the differential equation

$$\frac{d^2x}{dx^2} - 2\frac{dy}{dx} + y = e^x + x$$
6

2. a) Solve the differential equation :

$$\frac{d^2y}{dx^2} - 4y = x \sin x$$
7

   b) Solve the differential equation using method of undetermined coefficients:

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2 + \cos 2x$$
8

3. a) Solve the differential equation

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 10 \cos x$$

   given $y(0) = 1$ and $y'(0) = -1$.
8

P.T.O

b) Solve the differential equation using method of variation of parameters

$$\frac{d^2y}{dx^2} + a^2y = \sec ax$$

7

4. a) Find the volume of a parallelepiped whose sides are given by $\vec{A} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$, $\vec{B} = \hat{\imath} - \hat{\jmath} - 2\hat{k}$, and $\vec{C} = -\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$.

b) Calculate the Jacobian $J\left(\frac{x,y,z}{u,v,w}\right)$ of the transformation $x=u^2+w$, $y=u+v$ and $z = w^2 - u$.

c) If $\vec{v} = \vec{w} \times \vec{r}$, find whether $\vec{v}$ is solenoidal or not, where, $\vec{w}$ is a constant vector and $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$.

d) Find $\vec{\nabla}.(f(r)\vec{r})$, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$.

e) Find the directional derivative of a scalar function $\phi = (x^2 + y + z^2)^{-1/2}$ at the point $P(3, 1, 2)$ in the direction of the vector $yz\hat{\imath} + xz\hat{\jmath} + xy\hat{k}$.

3×5=15

5. a) Prove that

$$(\vec{A} \times \vec{B}).(\vec{C} \times \vec{D}) = (\vec{A}.\vec{C})(\vec{B}.\vec{D}) - (\vec{A}.\vec{D})(\vec{B}.\vec{C})$$

3

b) Evaluate

$$\vec{\nabla}^2 \left[\vec{\nabla}.\left(\frac{\vec{r}}{r^2}\right)\right]$$

where, $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

6

c) Evaluate

$$I = \oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$$

where C is the boundary of the region defined by $y^2 = x$ and $y = x^2$.

6

6. a) Verify Stoke's theorem for

$$\vec{A} = (2x - y)\hat{\imath} - yz^2\hat{\jmath} - y^2z\hat{k}$$

where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

10

b) Using Gauss Divergence theorem, prove that

$$\iiint_V \vec{\nabla}\phi \, dV = \iint_S \phi \, d\vec{S}$$

where V is the volume enclosed by the surface S. 5

7. a) Derive an expression of divergence of a vector field in orthogonal curvilinear coordinates. Express it in cylindrical coordinates. 6

b) Evaluate

$$\iiint_V (x^2 + y^2 + z^2) dV$$

where V is the volume of the sphere $x^2 + y^2 + z^2 = a^2$. 6

c) Define the Dirac Delta function and establish 3

$$\int_{-\infty}^{\infty} f'(x)\delta(x-a)\, dx = -f'(a)$$

This question paper contains 4 printed pages]

Roll No.

S. No. of Question Paper : 841

Unique Paper Code : 222101 G

Name of the Paper : **PHHT-101 : Mathematical Physics I**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : I

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *Five* questions in all including Q. No. 1 which is compulsory.

1. Do any *five* of the following : 5×3=15

(*a*) Find the derivative of $f(x, y, z) = x^2 - xy^2 - xe^x$ at P(1, 1, 0) in the direction of :

$$\vec{A} = \hat{i} - 2\hat{j} + 2\hat{k}.$$

(*b*) Prove that :

$$\nabla^2\left(\frac{1}{r}\right) = 0.$$

(*c*) Prove :

$$\iiint \frac{dV}{r^2} = \iint \frac{\vec{r} \cdot \hat{n}}{r^2} ds.$$

P.T.O.

(*d*) Let u_1, u_2, u_3 be the orthogonal coordinates. Prove that :

$$\vec{\nabla} u_p = h_p^{-1}, \qquad p = 1, 2, 3.$$

(*e*) What is the period of the function :

$$f(t) = 3 \sin(2\pi t) + 2 \cos(\pi t)$$

(*f*) If $x = u^2 + 2$, $y = u + v$, $z = \omega^2 - u$, find the Jacobian :

$$J\left(\frac{x, y, z}{u, v, w}\right).$$

(*g*) Evaluate :

$$\Gamma\left(\frac{-5}{2}\right).$$

2. (*a*) Prove that :

$$\vec{F} = \left(y^2 \cos x + z^3\right)\hat{i} + \left(2y \sin x - 4\right)\hat{j} + \left(3xz^2 + 2\right)\hat{k}$$

is a conservative force field. Find the scalar potential for $\vec{F}$. 5

(*b*) Prove the identity : 5

$$\vec{\nabla} \cdot \left(\vec{A} \times \vec{B}\right) = \vec{B} \cdot \left(\vec{\nabla} \times \vec{A}\right) - \vec{A} \cdot \left(\vec{\nabla} \times \vec{B}\right).$$

(*c*) Show that : 5

$$\vec{\nabla}\left(\frac{\vec{p} \cdot \vec{r}}{r^3}\right) = \frac{1}{r^3}\left(\vec{p} - 3\left(\vec{p} \cdot \hat{r}\right)\hat{r}\right).$$

3. (*a*) State and prove Gauss's Divergence theorem. 8

(*b*) Verify Green's theorem in the plane for :

$$\oint \left(3x^2 - 8y^2\right)dx + \left(4y - 6xy\right)dy,$$

where C is the boundary of the region defined by $y = \sqrt{x}$, $y = x^2$. 7

4. (*a*) Verify Stokes' theorem for :

$$\vec{V} = \left(3x - y\right)\hat{i} - 2yz^2\hat{j} - 2y^2z\hat{k},$$

where S is the upper half of the sphere $x^2 + y^2 + z^2 = 16$ and C is its boundary. 10

(*b*) Find the surface area of the plane $x + 2y + 2z = 12$ cut-off by the surfaces $x = 0$, $y = 0$ and $x^2 + y^2 = 16$. 5

5. (*a*) Derive an expression for divergence of a vector function $\vec{A}$ in orthogonal curvilinear coordinates system. 5

(*b*) Represent the vector $\vec{A} = x\hat{i} - 2z\hat{j} + y\hat{k}$, in cylindrical coordinates. Determine A_ρ, A_ϕ and A_z. 5

(*c*) Evaluate : 5

$$\iiint \left(x^2 + y^2 + z^2\right)dV,$$

where V is volume of the sphere :

$$x^2 + y^2 + z^2 = a^2.$$

6. (*a*) Expand as Fourier series, the function :

$$f(x) = \begin{cases} -k & -\pi < x < 0 \\ +k & 0 < x < \pi \end{cases}$$

Hence find the sum :

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \ldots\ldots\ldots$$

(*b*) If

$$f(x) = \begin{cases} x & 0 \le x \le \frac{\pi}{2} \\ \pi - x & \frac{\pi}{2} \le x \le \pi \end{cases}$$

Graph the function and express the function as cosine series.

7. (*a*) Prove that :

$$\beta(m, n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}.$$

(*b*) Evaluate :

$$\int_0^{\infty} \sqrt{x}\, e^{-x}\, dx.$$

(*c*) The radius of a cylinder is given as (2.0 ± 0.1) cm and height as (6.2 ± 0.2) cm. Fi

the volume of the cylinder and standard error in volume.

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1792 GC-3 Your Roll No...............

Unique Paper Code : 32221101

Name of the Paper : Mathematical Physics – I

Name of the Course : B.Sc. (Hons.) Physics : Choice-based Credit System

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt five questions in all.

1. (a) By calculating the Wronskian of the functions e^x, xe^x and e^{-x}, check whether the functions are linearly dependent or independent. (4)

(b) Solve the inexact equation

$$(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$$ (5)

(c) Solve the differential equation

$$\frac{d^2y}{dx^2} - y = e^x \cos x$$ (6)

2. (a) Solve the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} + \sin 3x$$ (8)

(b) Solve the differential equation using method of undetermined coefficients

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4$$ (7)

3. (a) Solve the differential equation

$$\frac{d^2y}{dx^2}+3\frac{dy}{dx}=1-9x^2$$

given $y(0) = 0$ and $y'(0) = 1$. (8)

(b) Solve the differential equation using method of variation of parameters.

$$\frac{d^2y}{dx^2}+a^2y = \operatorname{cosec} ax$$ (7)

4. (a) Find

$$\frac{d}{dt}\left(\vec{V}.\frac{d\vec{V}}{dt}\times\frac{d^2\vec{V}}{dt^2}\right)$$

where $\vec{V}$ is a function of t.

(b) Find the Jacobian $J\left(\frac{x,y,z}{u,v,w}\right)$ of the transformation

$u = x^2 + y^2 + z^2$, $v = x^2 - y^2 - z^2$ and $w = x^2 + y^2 - z^2$.

(c) If $\vec{v} = \vec{w}\times\vec{r}$, find whether $\vec{v}$ is irrotational or not, where, $\vec{w}$ is a constant vector and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

(d) Find $\vec{\nabla}\times\left(f(r)\vec{r}\right)$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

(e) Find the directional derivative of a scalar function $\phi = 2xz - y^2$ at the point (1, 3, 2) in the direction of $xz\hat{i} + yz\hat{j} + xy\hat{k}$. (3×5=15)

)2

(a) Prove that

$$\left(\vec{B}\times\vec{C}\right).\left(\vec{A}\times\vec{D}\right)+\left(\vec{C}\times\vec{A}\right).\left(\vec{B}\times\vec{D}\right)+\left(\vec{A}\times\vec{B}\right).\left(\vec{C}\times\vec{D}\right)=0$$

(b) Evaluate

$$\vec{\nabla}.\left[r\vec{\nabla}\left(\frac{1}{r^3}\right)\right]$$

where $r^2 = x^2 + y^2 + z^2$.

(c) Evaluate

$$I = \oint_C \left(3x-8y^2\right)dx + \left(4y-6xy\right)dy$$

where C is the boundary of the region bounded by $x = 0$, $y =$ ' $x + y = 1$.

(a) Verify Stoke's theorem when

$$\vec{F} = \left(2xy - x^2\right)\hat{i} - \left(x^2 - y^2\right)\hat{j}$$

where C is the boundary of the region enclosed by $y^2 = x$ and $x^2 =$

(b) Using Gauss Divergence theorem, prove that

$$\iiint_V \vec{\nabla}\times\vec{F}\, dV = \iint_S d\vec{S}\times\vec{F}$$

where V is the volume enclosed by surface S.

(a) Derive an expression of curl of a vector field in orthogonal curv coordinates. Express it in spherical coordinates.

Evaluate $\iiint_V (y^2 + z^2)\,dV$, where V is the volume bounded by the cylinde

$x^2 + y^2 = a^2$ and the planes $z = 0$ and $z = h$. (6

Define the Dirac Delta function and establish

$$\int_{-\infty}^{+\infty} f(x)\delta'(x)\,dx = -f'(0)$$

(3

his question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

6

. No. of Question Paper : **6671**

Jnique Paper Code : **32221101** HC

Jame of the Paper : **Mathematical Physics—I**

Jame of the Course : **B.Sc. (Hons.) Physics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all.

Question No. **1** is compulsory.

1. Do any *five* of the following : 5×3=15

(*a*) Two sides of a triangle are formed by the vectors :

$$\vec{A} = 3\hat{i} + 6\hat{j} - 2\hat{k} \quad \text{and} \quad \vec{B} = 4\hat{i} - \hat{j} + 3\hat{k}$$

Determine the angle between these two sides and length of the third side.

P.T.O.

his question paper contains 4+2 printed pages]

Roll No.

6

. No. of Question Paper : 6671

Jnique Paper Code : 32221101 HC

Vame of the Paper : **Mathematical Physics—I**

Vame of the Course : **B.Sc. (Hons.) Physics**

Semester : I

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all.

Question No. 1 is compulsory.

1. Do any *five* of the following : 5×3=15

(*a*) Two sides of a triangle are formed by the vectors :

$$\vec{A} = 3\hat{i} + 6\hat{j} - 2\hat{k} \text{ and } \vec{B} = 4\hat{i} - \hat{j} + 3\hat{k}$$

Determine the angle between these two sides and length of the third side.

P.T.O.

(b) Show that the area bounded by a simple closed curve is given by :

$$\frac{1}{2}\oint_{C} (x\,dy - y\,dx).$$

(c) If $\vec{a}$ is a constant vector, then prove that :

$$\vec{\nabla} \times \left(\vec{a} \times \vec{r}\right) = 2\,\vec{a}.$$

(d) Solve :

$$\iint_{R} \sqrt{x^2 + y^2}\, dx\, dy$$

where, R is the region bounded by the circle $x^2 + y^2 = 9$.

(e) Check whether the following functions are linearly independent or not :

$$e^x, x\,e^x.$$

(f) Solve the differential equation :

$$(b^2 + 2xy + y^2)dx + (x + y)^2 dy = 0.$$

(g) Form a differential equation whose solution is given by :

$$y = A\,e^{2x} + B\,e^{3x}.$$

(h) Solve :

(i) $\int_{0}^{5} \delta(x - \pi)\cos 2x\, dx$

(ii) $\int_{-2}^{2} \left[x^2 + \log x\right] \delta(x - 1)\, dx.$

2. (a) Find the constants 'a' and 'b' so that the surface $ax^2 - byz = (a + 2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point (1, –1, 2). 4

(b) If $\vec{A} = r^n \vec{r}$, then find the value of n for which $\vec{A}$ is solenoidal. 5

(c) Prove that : 6

$$\vec{\nabla} \cdot \left[r\, \vec{\nabla}\left(\frac{1}{r^3}\right)\right] = \frac{3}{r^4}$$

where, $r = \sqrt{x^2 + y^2 + z^2}$.

3. (a) Prove that : 6

$$\vec{A} \times \left(\vec{\nabla} \times \vec{A}\right) = \frac{1}{2}\vec{\nabla} A^2 - \left(\vec{A} . \vec{\nabla}\right)\vec{A}.$$

(b) Evaluate $\iint_S \left(\vec{A} . \hat{n}\right) dS$, where :

$$\vec{A} = y\hat{i} + 2x\hat{j} - z\hat{k}$$

And,

S is the surface of the plane, $2x + y = 6$ in the first octant cut-off by the plane, $z = 4$. 9

4. (a) Prove that : 5

$$\oiint_S r^5 \hat{n}\, dS = \iiint_V 5r^3 \vec{r}\, dV$$

where, simple closed surface S encloses volume V.

(b) Write the mathematical form of Gauss's Divergence theorem and hence verify it for $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$, where S is the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$. 1,9

5. (a) Evaluate :

$$\iiint_V (2x + y)\,dV,$$

where, V is the closed region bounded by the cylinder $z = 4 - x^2$ and the planes, $x = 0,\ y = 0,\ y = 2,\ z = 0$. 6

(b) Derive an expression for curl of a vector field in orthogonal curvilinear coordinates. Express it in cylindrical coordinates. 7,2

6. Solve the differential equations :

(a) $(x^2y - 2xy^2)dx - (x^3 - 2x^2y)dy = 0$ 7

(b) $(D^2 + 1)y = \text{cosec } x \quad \left(D = \frac{d}{dx}\right)$. 8

7. (a) Solve the differential equation : 7

$$(D^2 - 6D + 8)y = (e^{2x} - 1)^2.$$

(b) Using method of variation of parameters, solve the differential equation : 8

$$(D^2 + 4)y = x \sin 2x.$$

8. (*a*) Solve the differential equation :

$$(D^2 - 4D + 3)y = xe^{2x}.$$

(*b*) Using method of undetermined coefficients, solve th

differential equation :

$$(D^2 - 1)y = e^x + 2x.$$

2

This question paper contains 4 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper : 102 I

Unique Paper Code : 32221101

Name of Paper : Mathematical Physics - I

Name of Course : B.Sc. (Hons.) Physics

Semester : I

Duration : 3 hours

Maximum Marks : 75

07 DEC 2018

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt five questions in all.
Question No. 1 is compulsory.

1. Do any *five* questions :

   (a) Solve: $\frac{dy}{dx} = (1+x^2)(1+y^2)$.

   (b) By calculating the Wronskian of the functions e^x, e^{-x}, and e^{-2x} check whether the functions are linearly dependent or independent.

   (c) Find the area of the triangle with vertices P(2, 3, 5), Q(4, 2–1), and R(3, 6, 4).

   (d) Find the unit vector normal to the surface $x^2 + y^2 + z^2 = 4$ at the point $(1, \sqrt{2}, -1)$.

   (e) Show that :

   $$\oint\!\!\oint_S \left(\vec{\nabla} r^2\right) \cdot \vec{dS} = 6V$$

   where S is the closed surface enclosing the volume V.

P.T.O.

(f) Evaluate :

$$\iint_R \sqrt{x^2 + y^2}\, dx\, dy$$

(g) Verify that :

$$\int_{-\infty}^{+\infty} \delta(a-x)\delta(x-b)\, dx = \delta(a-b)$$

(h) Form a differential equation whose solutions are e^{2x} and e^{3x}. 5×3=15

2. (a) Solve the inexact equation :

$$y(1+xy)\, dx + x(1+xy+x^2y^2)\, dy = 0.$$ 5

(b) Solve the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x \cos x.$$ 4

(c) Using method of undetermined coefficients, solve the differential equation :

$$\frac{d^2y}{dx^2} + 4y = 2\sin 2x.$$ 6

3. (a) Solve the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = x^2 + 5.$$ 9

(b) Solve the differential equation using method of variation of parameter

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2 e^{2x}.$$ 6

4. (a) Show that

$$\left[\left(\vec{A}\times\vec{B}\right)\times\vec{C}\right]\times\vec{D}+\left[\left(\vec{B}\times\vec{A}\right)\times\vec{D}\right]\times\vec{C}+$$
$$\left[\left(\vec{C}+\vec{D}\right)\times\vec{A}\right]\times\vec{B}+\left[\left(\vec{D}\times\vec{C}\right)\times\vec{B}\right]\times\vec{A}=0. \quad 6$$

(b) Show that :

$$\vec{F}=\left(y^2\cos x+z^3\right)\hat{i}+\left(2y\sin x-4\right)\hat{j}+\left(3xz^2+2\right)\hat{k}$$

is a conservative force field and then evaluate

$$\int_C \vec{F}\cdot\vec{dr}$$

where C is any path from (0, 1, –1) to $\left(\frac{\pi}{2}, -1, 2\right)$. 9

5. (a) If $\vec{a}$ is a constant vector and $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ then prove that :

$$\text{curl}\left(\frac{\vec{a}\times\vec{r}}{r^3}\right)=\frac{3\left(\vec{a}\cdot\vec{r}\right)\vec{r}}{r^5}-\frac{\vec{a}}{r^3}. \quad 7$$

(b) Evaluate :

$$\iint_S \vec{A}\cdot\hat{n}\,dS$$

where $\vec{\text{A}}=18z\hat{i}-12\hat{j}+3y\hat{k}$ and S is the part of the plane $2x+3y+6z=12$ located in the first octant. 8

6. (a) Evaluate :

$$\oint_C \left(y-\sin x\right)dx+\cos x\,dy$$

(i) directly

(ii) using Green's theorem in the plane, where C is the boundary of a triangle enclosed by the lines $y = 0$, $x=\frac{\pi}{2}$, and $y=\frac{2}{\pi}x$. 10

(b) Verify that :

$$\nabla^2 r^n = n(n+1)r^{n-2}.$$ 5

7. (a) Verify divergence theorem for

$$\vec{F} = (x^2 - yz)\hat{i} + (y^2 - xz)\hat{j} + (z^2 - xy)\hat{k}$$

taken over the rectangular parallelopiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$. 10

(b) Express the position and velocity of a particle in cylindrical coordinates. 5

8. (a) Derive an expression for the divergence of a vector field in orthogonal curvilinear coordinate system. 10

(b) Evaluate Jacobian $J\left(\frac{x, y, z}{u_1, u_2, u_3}\right)$ for the transformation from rectangular coordinate system to spherical coordinate system. 5